Compute airlift on a hot air balloon

Prelude
This is a simplified calculation to compute the lift capacity of a hot air balloon. The balloon in question is “UFO – Hot Air Balloon” from Banana Bending Company, see www.BananaBending.net for more information. The balloon looks like the picture to the right. I and my two boys, Alexander and Anders did an experiment with one of these balloons in the Gothenburg (Sweden) area. It was clearly very amusement and fun. Afterwards I did some calculations about the physics around this event. One question I had was if the balloon had the lift capacity to bring along some electronics to telemetry the position height etc. Make a note that some of the measurements are rather rude and therefore are the calculation an estimate as to best of knowledge.

Step 1, Volume of Balloon
To be able to calculate the lift capacity of the balloon the first steep is to compute the total volume of the balloon. The way I did this was by using a picture of the balloon and measures the height of it. There after scale all other measurement in scale to this. I used MS Visio to do this and end up with the drawing as below. Note all numbers are in centimeters [cm].

Now we can calculate the volume by slicing up the balloon in a number of cones, numbered S1 to S12 in the drawing above. For each cone we calculate the volume and add up all the slices to get the total volume.
The formula to calculate the volume, area etc for a cone is as below:

**Volume of balloon**

Frustum of right circular cone:

- Ra = Radius of lower base
- Rb = Radius of upper base
- s = Slant height
- h = Altitude
- V = Volume
- T = Total surface area
- S = Lateral surface

1. \[ T = \Pi * \left( Ra^2 + Rb^2 + (Ra + Rb)\sqrt{(Ra - Rb)^2 + h^2} \right) \]

2. \[ S = \Pi * (Ra + Rb) * \sqrt{(Ra - Rb)^2 + h^2} \]

3. \[ V = \frac{1}{2} * \Pi * h * (Ra^2 + Rb^2 + Ra * Rb) \]

Now we put this into an Excel calculus sheet and we get the following:

**Balloong Volume & Area**

- Ra Radius lower base
- Rb Radius upper base
- h Altitude (height)
- T= Total surface area
- S= Lateral surface area
- V = Volume

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<th>Db</th>
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<th>h</th>
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| Sum: | 0,50 | 2,30 | 12,36 |
Balloon Volume [m2] 0.50

We get that the total volume of the balloon is 0.5 square meters [m2]

Step 2, Lift Force of the Balloon
The next step is now to calculate the lift force of the balloon. This is a rather straight forward calculation. What we need to do is to compute the change in air density when the air inside the balloon is heated up. First we calculate the air density of the balloon based on the ambient temperature and then we redo the calculation for the heated air inside the balloon. Knowing the volume of the balloon then simply get the lift force by taking the difference of the two computations and multiply this by the volume of the balloon. So here we go:

1. \( K = C + 273.15 \)
   \( K = \text{degrees Kelvin} \)
   \( C = \text{degrees Celsius} \)

2. \( P = 100000 \times 0.001 \times mB = 100 \times mB \)
   \( P = \text{Pascal [N/m2]} \)
   \( mB = \text{Millie Bar} \)

2. \( \rho_d = \frac{P}{(R \times T)} \)
   \( \rho_d = \text{Density of air in kg/m}^2 \)
   \( P = \text{Pressure in Pascal's of air} \)
   \( R = \text{Specific gas constant for air, J/(Kg * K) (J = Joule), 287.05 for dry air} \)
   \( T = \text{Temperature in Kelvin's} \)

3. \( F_l = \rho_d^2 - \rho_d^1 \times V \)
   \( F_l = \text{Lift force in kg} \)
   \( \rho_d^1 = \text{Density of air per m2 in ambient temperature} \)
   \( \rho_d^2 = \text{Density of air per m2 of hooted air inside balloon} \)
   \( V = \text{Volume of balloon} \)

4. \( F_l = \frac{(100 \times mB/(287.05 \times (C_2 + 273.15))) - (100*mB/(287.05 \times (C_1 + 273.15))) \times V}{\text{Lift force in kg of balloon}} \)
   \( mB = \text{Air pressure in mille bar} \)
   \( C_2 = \text{Temperature of air inside balloon in Celsius degrees} \)
   \( C_1 = \text{Temperature of surrounding air in Celsius degrees} \)
   \( V = \text{Volume of balloon in m2 (Cubic meters)} \)

We did measure the barometric pressure and outdoor temperature in conjunction with the flight test. We did also measure the inside temperature of the balloon just before releasing it as well as the lift force. The later was measure by a dynamometer, see picture the right. This was graded from 0 to 100 grams. I did a calibration of before use and it seemed to be accurate within +/- 2 grams.

To measure the temperature we used a cheap electronic thermometer with a digital LCD display that you can buy more or less everywhere today. The thermometer has a temperature sensor connected to the meter with a two meter cable. To avoid burning the cable it was placed inside a brass tube with a diameter of 7 mm. The sensor was then put into the center of the balloon after the ignition of the burner. We waited about one minute before taken the measurement to give sufficient time for the air to be heated up.

We toke the following notes:
1. Weight of balloon: 108 grams [g]
2. Barometric pressure: 960 mille Bar [mB]
3. Outdoor temperature: 4 Celsius degrees [C]
4. Balloon heated air temperature: 81 Celsius degrees [C]
5. Lift force of balloon measured with dynamometer: 18 grams [g]

**Remark**
The measurement of the lift force with the dynamometer is very uncertain. The uncertainty was caused by the rather windy situation at the moment of the ignition and release of the balloon. It was difficult to judge if I was measure the lift force or the wind force on the balloon. I have to redo this in another experiment on a calm day.

Now, having the parameters above, we can calculate the lift force of the balloon. Using formula four above we get that the lift force of the balloon is:

\[ Fl = \frac{100 \times 960}{(287,05 \times (4 + 273,15))} - \frac{100 \times 960}{(287,05 \times (81 + 273,15))} \times 0,50 \]

We get that \( Fl = 0,1315 \text{ Kg} \)

From this we have to subtract the weight of the balloon itself. The weight of the balloon was measured to be 0,108 grams. So we get that the net lift force is:

**Net Lift Force = 0,1315 – 0,108 = 0,0235 Kg or 23,5 grams [g]**

Not so much marginal to put some extra telemetry devices to the balloon for more measurements, a real challenge! The net lift force is also pretty good in agreement with the measurement done with the dynamometer that was done just before releasing the balloon.

**Postlude**
We estimate the fuel time to about six to seven minutes. The fuel consists of paper strings mixed with paraffin. We did a test using a hot air gun to heat up the balloon to test it. The hot air gun was rated with a nominal power of 1400 Watts. This could barely fill the balloon and the lift force was close to zero or negative. Obviously the heater power much exceed 1400 Watts.

After releasing the balloon it took of quite speedy, after about five minutes it did disappear into the clouds. After coming back home I looked up the fly weather and found the following below. You can find this information at:
Decoding this gives that the wind speed is 5 knots with a wind direction of 120 degrees. The lower cloud base is at 1500 foot, which is about 500 meters.

Now, the balloon did disappear into the clouds after about 4 minutes, this gives a climb rate of about 500/(4*60) = 2 meters per second. Further more the balloon did drift of with about 5 knots per seconds. Note that one (1) international knot = 1 nautical mile per hour (exactly), this is the same as 0.514 meters per second. So in our case we hade 5 * 0,514 m/s that is 2,57 m/s. This gives that the ground distance to the balloon was 4 * 60 * 2,57 = 617 meters.

The total distance to the balloon (Td) was before it disappeared into the clouds about:

$$ Td = \sqrt{617^2 + 500^2} $$

Td ≈ 794 meters

If we estimate the fuel time to 7,5 minutes the total distance that the balloon will make before descending again is roughly:

Height: 500 * 7,5/4 = 938 meters
Ground distance: 1157 meters
Td = 1489 meters

.End of stack!